原著

An Analysis of the External Work of the Left Ventricle and the Energy Consumption at Resistive Arterial Bed during the Optimally Controlled Hemodynamical State.

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Abstract

The external work of the left ventricle and the energy consumption at resistive arterial bed have been calculated under the condition of the optimality in circulatory system that minimizes the external work of ventricle, potential energy and the rate of change in arterial blood flow. The energy consumed at resistive arterial bed and the external work of ventricle increased linearly with fluid resistance. At a given magnitude of the fluid resistance, they decreased with arterial compliance while they increased with aortic valvular resistance. The external work increased with the shortening of the ejection duration and with the increase of the stroke volume. The changes in the weighting coefficients relating to the external work and the potential energy did not affect the energy consumed at arterial bed nor the external work of ventricle. Thus the changes in the afterload impedance, in the ejection duration and in the stroke volume are more important on the energy consumption at arterial bed and the external work of ventricle than the relative ratio of the minimization of the external work and the potential energy of ventricle. Since present theoretical method enables us to calculate the energy consumption and the external work of ventricle without actual measurement, it is available for clinical application.

Introduction

Many indices have been proposed to evaluate the total behavior of cardiovascular system. Most of them, however, have not been based on control theory nor referred to any biological control strategy relating to the economicity that minimizes the consumption of energy while maximizes the efficiency of blood perfusion. Among many candidates, the concept of optimal control seems to be the most natural one. Intuitively, to maintain the circulatory homeostasis, cardiovascular system must satisfy the requirements from organs to supply sufficient blood flow and from ventricle to rest as possible to save oxygen (1). Previous our works (2,3) have shown an existence of the optimal control in the cardiovascular system. In the paper, for the quantitative evaluation on cardiovascular function, we analyzed the external work and the energies consumed at arterial bed in relation to changes in system parameters under the optimal control.

Method

To describe the working ability of cardiovascular system, the optimal control theory always sets up a performance function. We defined that the cardiovascular system is operating optimally only

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when the following performance function is minimized.

$$J=\int_{0}^{Te}\left[(dIa(t)/dt)^{2}+\alpha P(t)Ia(t)\right]dt+\beta P(Te) \quad \cdots(1)$$

where Ia (t) is aortic flow rate (ml/sec), P (t) is ventricular ejection pressure (mmHg), Te is the end ejection time (sec). The first term is the second power of the rate of change in arterial flow. Minimization of this term results in a stable blood flow to organs. The second term expresses the external work of ventricle (Fig 1: the shaded area). The α is the weighting coefficient (ml/ (mmHg s^3)) related to the external work of ventricle. Minimization of this term concerns the saving of external work of ventricle. An increase in α means a stronger minimization is imposed on the external work of ventricle. The third term is proportional to the potential energy of ventricle (4) (Fig 1) which relates to an increase in the elastance of ventricle at isovolumic contraction. The β is the weighting coefficient (ml^2/ (mmHg s^3)) related to the potential energy. The sum (pressure volume area:PVA) of these two terms can approximate the total work done by a ventricular contraction (4). This area is highly proportional to the oxygen consumption of ventricle (1,4). These weighting coefficients represent the relative magnitude of minimization of these terms. The physiological significance of this performance function relates to the Homeostasis. From view point of evolution, the cardiovascular system must have conquested inherent contradictions to maintain Homeostasis. The ventricle is supposed to work as little as possible to save oxygen consumption although it may result in poor organ perfusion. The organs, on the other hand, must require stable

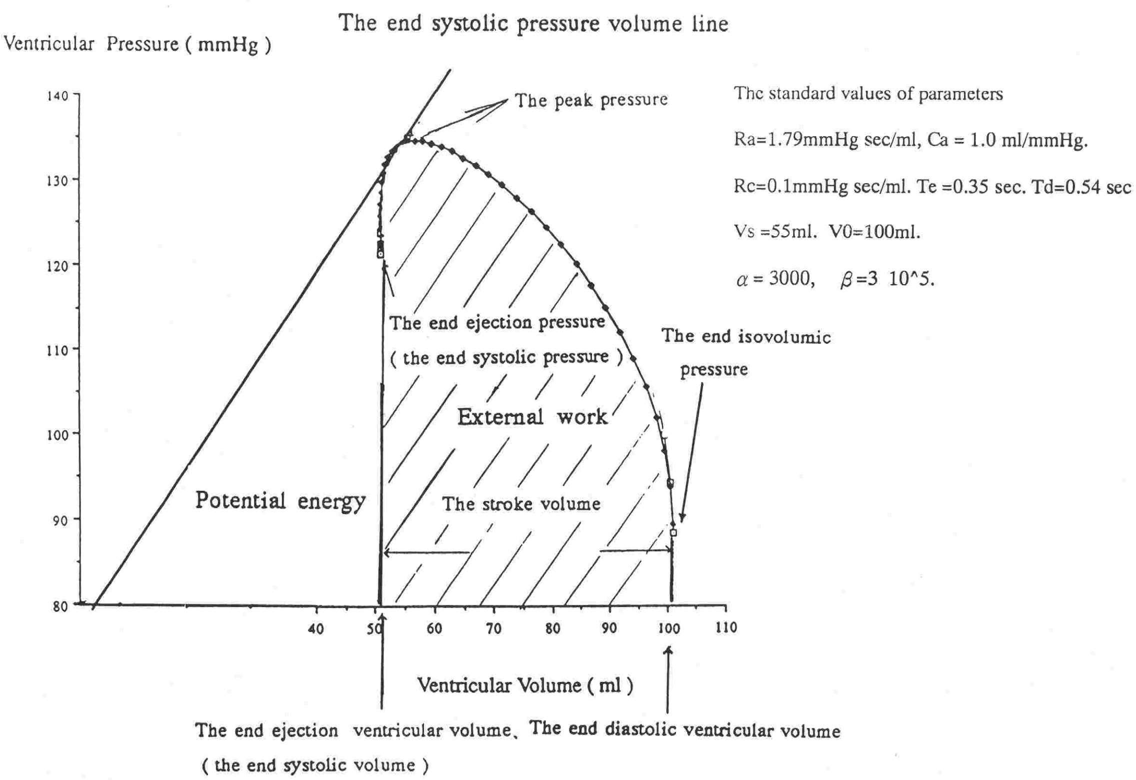

Fig 1 The ppressure volume loops expresses the external work and the potential energy of a given ventricular contraction. The curve during diastolic phase is neglected.

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blood supply even though ventricle is enforced to work much. Thus minimizing only the work of ventricle nor only the rate of change in flow is insufficient to achieve circulatory Homeostasis. Consequently, satisfying those contradictional demands from ventricle and organs simultaneously is indispensable to maintain the homeostasis and its engineering expression can be reduced to the optimality. The arterial system is expressed by the Wind Kessel model (5).

$$P(t) = Rc\, Ia(t) + Pa(t) \quad \cdots (2)$$

$$Ia(t) = Pa(t)/Ra + Ca\, dPa(t)/dt \quad \cdots (3)$$

$$V(t) = V0 - \int_0^{Te} Ia(t)\, dt \quad \cdots (4)$$

where V (t) is ventricular volume (ml), Rc is the aortic valvular resistance (mmHgsec/ml), Ra is arterial resistance (mmHgsec/ml), Ca is aortic compliance (ml/mmHg). V0 is the end diastolic volume (ml). The initial and the boundary conditions for aortic flow are

$$Ia(0) = Ia(Te) = 0 \quad \cdots (5)$$

For the simplicity of analysis, we define the end ejection timing as the exact instance at which the aortic valve closes. During diastolic phase, the transitional condition is

$$Pa(0) = K\, Pa(Te) \cdots\cdots (6).\ K = \exp(-Td/(Ra\, Ca))$$

where Td is the diastolic duration. An integral constraint for aortic flow rate is

$$Vs = \int_0^{Te} Ia(t)\, dt \quad \cdots (7)$$

where Vs is the stroke volume. This constraint is necessary because the Ia (t) itself has an inherent limitation. To obtain the optimal control of the cardiovascular system, the present probrem has been reduced to minimize the performance function (equation (1)) with conditions (2) - (7). We have utilized the optimal control theory (6) to convert this biological problem to a formal mathematical problem (2,3). A summary of mathematical expansion is given in APPENDIX. In the following results, once an optimal arterial pressure Pa (t) has been determined, aortic flow Ia (t) and ventricular pressure P (t) can be obtained by equation (2), (3). The optimal solution is given at individual changes in alternating the system parameters such as Ra and Ca.

The calculation of energies

Once the optimal pressure and flow have been determined, the external work and energies were calculated by numerical integration. We have calculated two kinds of energies. The first one was the hydro-mechanical energy : P (t) Ia (t) dt approximating the external work of ventricle and Pa (t) Ia (t) dt as the energy consumed at arterial bed. Another one was the electrically equivalent energy which is consumed at Ra : $Pa(t)^2/Ra$ dt and at Rc : $Ia(t)^2$ Rc dt. The calculations of pressure and flow were achieved only when the performance function has been minimized by strict mathematical expansion after Pontoryagin (6). Thus, a given set of system parameters and the weighting coefficients affords a single solution;they can produce only one optimal pressure and flow curves.

Results

1) Ability to reproduce physiological pressure and aortic flow curves. (Fig 2)

Firstry, to verity the validity of our theory, we showed ventricular pressure and aortic flow curves obtained from equation (2) and (3) by alternating Ra, Ca and Rc while Te and Vs were kept constant. The standard values of system parameters were shown in Table 1 which were obtained from normal human subject. With an increase in Ra (Fig 2-a), the peak pressure increased while the peak flow rate decreased. With a reduction of Ca (Fig 2-b), the peak pressure increased while the diastolic pressure decreased and the peak flow rate increased. These were not strange results because Vs and Te were set to be constant. We had discussed about the

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Fig 2-a　　Fig 2-b　　Fig 2-c

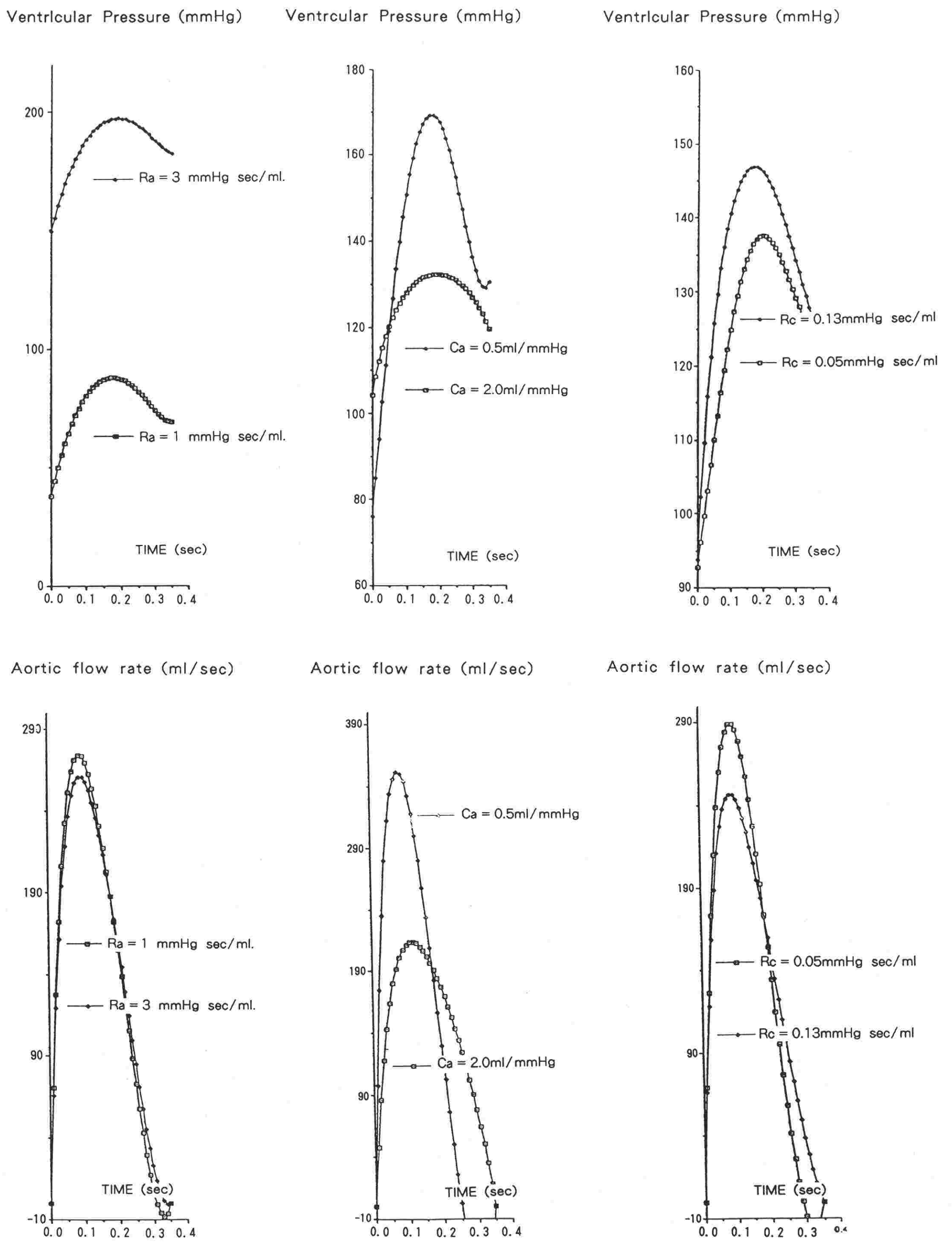

Fig 2 Upper part is recontracted ventricular pressure and lower part is aortic flow curves. Fig 2–a shows the effects of Ra. Fig 2–b describes the effects of Ca. Fig 2–c expresses the effects of RC.

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effect of Ca in detail in previous our papers (2,3). With an increase in Rc (Fig 2-c), the peak pressure increased while the peak flow rate decreased. The reactions of pressure and flow curves to changes in after load were consistent with physiological observations (7).

2) The external work with changes in the system parameters. (Fig 3)

At a given magnitude of resistance Ra, the external work was calculated by alternating Ca (Fig 4-a), Rc (Fig 4-b), Te (Fig 4-c) and Vs (Fig 4-d), while all other parameters were set at the standard level. At a given magnitude of Ra, the external work increased with Rc and Vs while it decreased with Ca and Te.

3) The energies developed at resistive components. (Fig 4).

The energies and the external work were calculated at different level of Ra (Fig 4-a), Ca (Fig 4-b) and Rc (Fig 4-c). At any situation, the external work, (P (t) Ia (t) dt), was the greatest and lesser degree the hydraulic energy Pa (t) Ia (t) dt. Electrically equivalent energies at Ra and Rc were considerably smaller than them. With an increase in Ra, the external work, Pa (t) Ia (t) dt, and the energy at Ra increased linearly while the energy at Rc did not change. All kinds of energies decreased with Ca. With increases in Rc, the external work and the energy at Rc increased linearly while Pa (t) Ia (t) and the energy at Ra did not change.

4) Effects of changes in weighting coefficients α and β (Fig 5).

Changes in the weighting coefficients, α and β indicate alterations of relative magnitude of minimization of the external work and the potential energy. We should be careful that an increase in weighting coefficient does not mean an increase in related quantity. All kinds of energy did not change even though α and β were altered extensively.

Discussion

Present analysis has been confined on the external work of ventricle and the energies consumed at arterial bed under the optimal control minimizing the second power of the rate of change in organ perfusion and pressure volume area of a given ventricular contraction. The ability of the optimal control theory to simulate actual ventricular pressure and aortic flow curves have been already shown in our previous reports (2,3). The most striking point of the present investigation is the ability to obtain the external work of ventricle and the energies at arterial system without measuring flow rate nor pressure volume loop because once a ventricular pressure could be obtained by the simulation based on the optimal control theory, aortic flow and related variables can be calculated automatically.

The linear increase in the external work of ventricle with resistance (Fig 3) is consistent with physiological considerations. The increase in compliance Ca particularly at its smaller range (Fig 4-b) has reduced the energies which effect is independent of Ra (Fig 3-a). This means that an elastic compliant tube would absorb pressure impact to arterial wall which in turn reduces the energy consumption and the work of ventricle. The shortening of ejection duration (Fig 3-c) required an increase in the external work because the cardiovascular system has to eject out a given amount of stroke volume during squeezed duration. An increase in stroke volume (Fig 3-d) also required an enhancement of the external work of ventricle. The influences of Te and Vs heavily depend on the magnitude of Ra even though the cardiovascular system has been operated under the optimality. Thus reduction of resistance is quite effective in saving the external work particularly at an elevated heart rate and at an increased stroke volume.

Although the electrically equivalent and hydro-mechanical energies (Fig 4) showed a similar pattern, the changes of hydro-mechanical ones are more rapid and larger in magnitude. Thus the hydro-mechanical energies would have a higher sensitivity to alteration of after load than electrical one. Considering the invariability of energies due to changes in weighting coefficients (Fig 5), the relative magnitude of minimization dose not play an important role in the production of the external work

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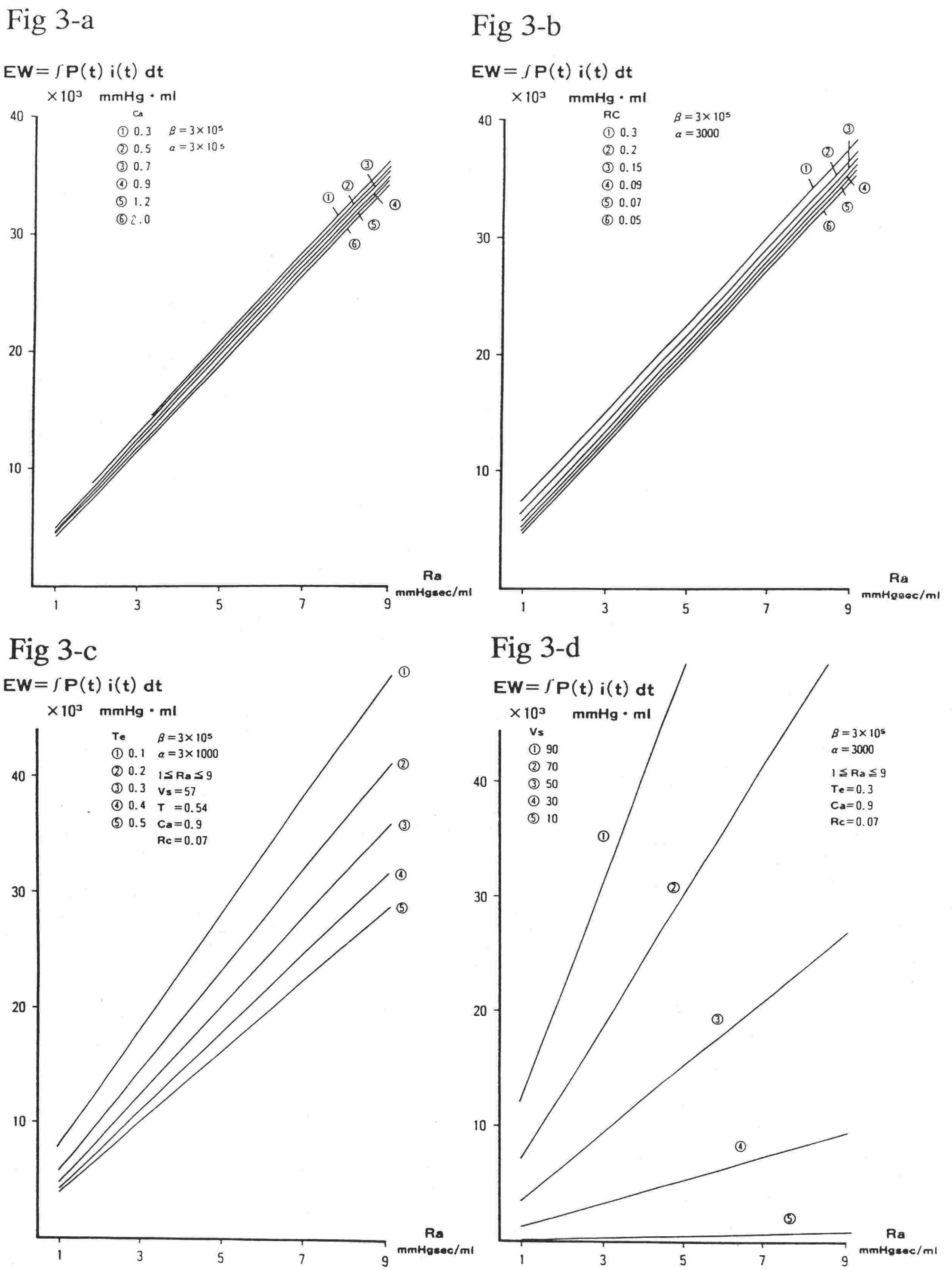

Fig 3 The effects of changes in Ca (Fig 3–a), in RC (Fig 3–b), in Te (Fig 3–c) and in Vs (Fig 3–d) on the external work as parametric changes.

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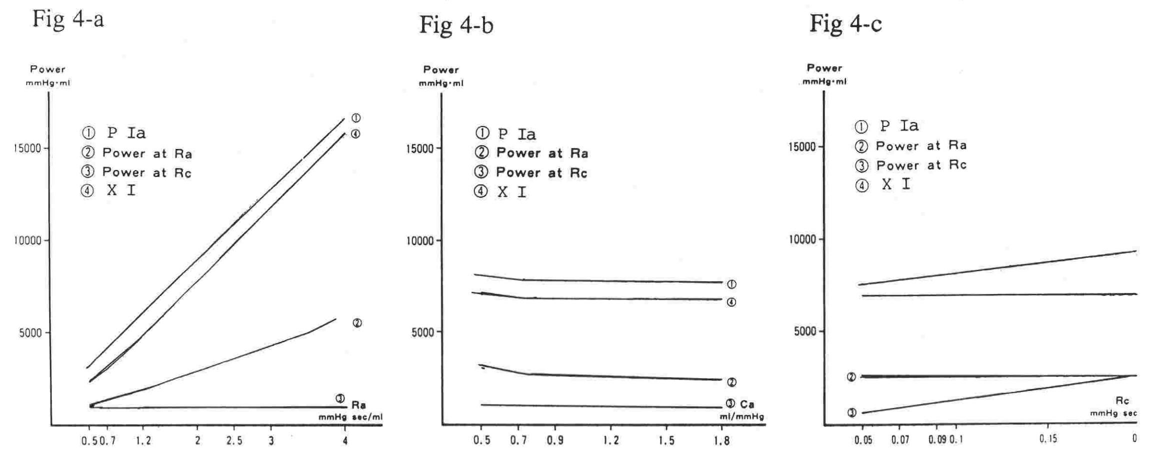

Fig 4 The changes in energies with Ra (Fig 4–a), with Ca (Fig 4–b) and with RC (Fig 4–c).

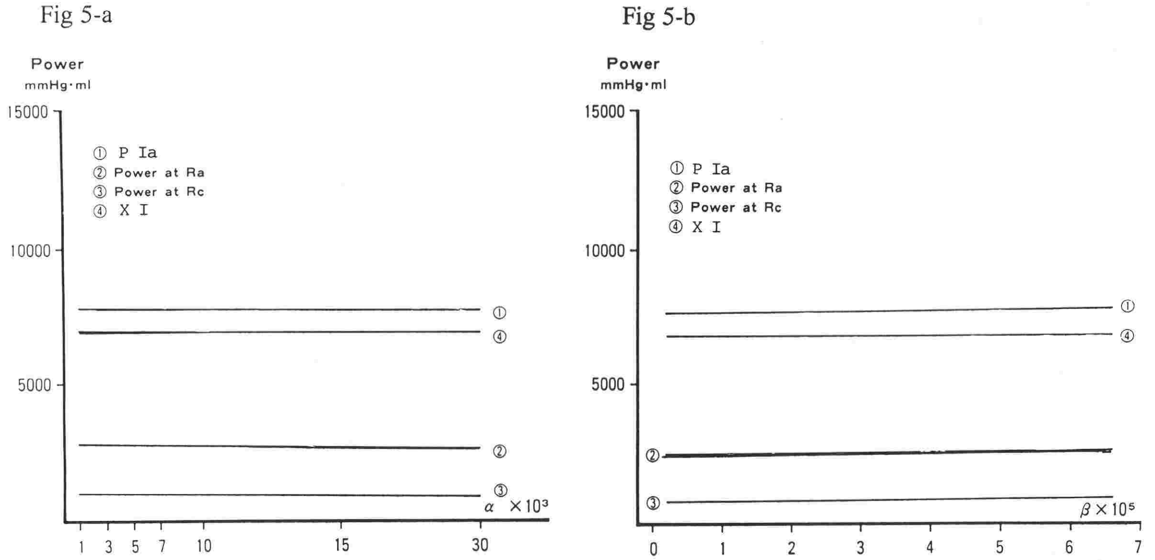

Fig 5 The changes in energies with alternating the weighting coefficients α (Fig 5–a) and β (Fig 5–b).

and the energies as long as the cardiovascular system has been driven under the optimality described by the performance function.

In summary, following conclusions were obtained.

1) Under the optimal control of cardiovascular system minimizing the second power of the rate of change in arterial flow and the pressure volume area of ventricle, the external work of ventricle and the energies consumed at resistive components of arterial system increased linearly with resistance and decreased hyperbolically at smaller range of compliance.

2) At a given Ra, the external work increased with shortening the ejection duration and with stroke volume which heavily depend on the resistance.

3) The relative magnitude of minimization of the

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external work and the potential energy of ventricle did not influence the energies and the external work of ventricle.

Appendix

By setting state variables [X1, X2, X3] and the control variable U (t) as

X1 (t) = Pa (t), X2 (t) = V (t), X3 (t) = Ia (t) and U (t) = d Ia (t) / dt which is the optimal control of the system. Then state equations are represented as

dxl (t) / dt = -x1 (t) / (RaCa) + x3 (t) / Ca. dx2 (t) / dt = - x3 (t). dx3 (t) / dt = U (t)

Hamiltonian H is formulated as

H [X, U, P] = α (Rc X3 (t)^2 + X1 (t) X3 (t)) + U (t)^2 + P1 (t). (-X1 (t) / (Ra Ca) + X3 (t) / Ca) - P2 (t). X3 (t) + P3 (t)·U (t)

where P1 (t), P2 (t) and P3 (t) are the co-state variables. The optimal control is obtained by differentiating H [x, u, p] with respect to U (t), as

dH [x, u, p] / dU (t) = 0.

The differential equations for co-state variables are obtained by

dp1 (t) / dt = - dH/dx1. dp2 (t) / dt = -dH/dx2. dp3 (t) / dt = -dH/dx3

The boundary conditions are represented as

x2 (0) = Vo, x3 (0) = 0, x1 (0) - Kx1 (Te) = 0, x2 (Te) = V0 - Vs. x3 (Te) = 0, K p1 (0) - p1 (Te) = - β

These 6 simultaneous differential equations are solved by multiple shooting method.

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